Nonlinear model-based control of combustion timing in premixed charge compression ignition

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Abstract
One of the major challenges in the control of advanced combustion modes, such as premixed charge compression ignition, is controlling the timing of the combustion event. A nonlinear model-based controller is outlined and experimentally shown to be capable of controlling the engine combustion timing during diesel premixed charge compression ignition operation on a modern diesel engine with variable valve actuation by targeting the desired values of the in-cylinder oxygen mass fraction and the start of injection. Specifically, the experimental results show that the strategy is capable of controlling the start of combustion and the intake oxygen mass fraction to within \(1^{\circ}\) crank angle and 1\% respectively. A stability analysis also demonstrates that this control strategy ensures asymptotically stable error dynamics.

Keywords
Engine control systems, engine emission control, engine valvetrains, heavy-duty diesel engines, low-emission engines

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Introduction
Engine developers continually balance the tightening emissions legislation\(^1\) with requirements to maximize the fuel efficiency. In modern diesel engines, aftertreatment systems are deployed to reduce the engine-out emissions to the legislated tailpipe-out levels.\(^2\)\(^-\)\(^4\) An alternative to this approach is the use of advanced combustion modes including diesel premixed charge compression ignition (PCCI), homogeneous charge compression ignition (HCCI), and low-temperature combustion (LTC) which offer the potential of reduced emissions while maintaining a high engine efficiency.\(^5\)\(^-\)\(^11\)

In particular, PCCI can be achieved on diesel engines by lengthening the time period between the start of injection (SOI) and the start of combustion (SOC) such that there is sufficient time for the fuel and air to premix prior to combustion. Combustion then occurs in one rapid event at a rate governed by the chemical kinetics of the combustion reaction. This leads to lower local temperatures and lower equivalence ratios and shifts operation to regions where particulate matter and nitrogen oxide emissions are dramatically reduced,\(^12\) as demonstrated in Figure 1. While dramatic reductions in the emissions can be achieved through this combustion strategy, the lack of a direct combustion trigger has previously limited the widespread adoption of diesel PCCI and other advanced combustion modes.

Additionally, many of these advanced combustion techniques are enabled through the use of flexible valvetrains, as will be considered in this work.\(^13\)\(^-\)\(^19\) The use of variable valve actuation (VVA) allows alterations in the effective compression ratio (ECR) of the engine to be made, thereby enabling changes to the engine’s gas exchange processes to be carried out.

In PCCI, the SOI will be followed by an ignition delay period which is significantly affected by the in-cylinder conditions. If the ignition delay period is not sufficiently long, there will not be adequate time for complete mixing of the air and fuel prior to combustion. While control of the SOI alone can provide some

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combustion phasing control, combustion phasing control which utilizes the SOI as its sole input will not be robust to changes in the in-cylinder contents, the temperature, and the pressure. To place the combustion event accurately, proper control of the nonlinear gas exchange processes as well as control of the fuel injection timing are essential in order to maintain the required in-cylinder temperatures necessary to achieve such an LTC mode.

Several key studies have investigated control of the gas exchange processes through the use of a nonlinear model-based controller such as that utilized in this work. Previous efforts included a paper by Jankovic et al. in which a nonlinear model-based controller was developed to regulate the air-to-fuel ratio and the exhaust gas recirculation (EGR) fraction on a diesel engine equipped with a variable-geometry turbocharger (VGT) and an EGR valve. The controller was experimentally validated against engine data containing reference set-point steps. Operations of the engine under typical diesel diffusion combustion and advanced combustion modes were not considered. Jung and Glover in which a proportional–integral controller was developed by three of the present authors and co-workers. The controller was experimentally validated against engine data containing the EGR valve and VGT actuator step changes and the engine load ramps. The controller took into account the advanced combustion modes but did not control or estimate the combustion phasing.

Unlike these previous efforts, this work uses a nonlinear model-based controller to provide control of the gas exchange and fueling processes and to achieve tracking of a desired SOC target in diesel PCCI combustion. The controller utilizes the VGT actuator to control the intake manifold oxygen fraction to a desired value and additionally modifies the start of injection commanded by the engine electronic control module (SOI_{ECM}) to achieve the desired SOC. Feedback for the oxygen fraction controller is provided by an oxygen fraction estimator developed by three of the present authors and co-workers. Stability of the PCCI combustion timing controller is shown through application of the Lyapunov theory. The controller is designed to track the desired intake manifold oxygen fraction to within 1% oxygen with asymptotic stability. The desired SOC is achieved by inverting a PCCI combustion timing model to calculate the necessary SOI command. The desired SOC is controlled to within 1° crank angle (CA). The controller is validated on an engine utilizing high-pressure cooled EGR, variable-geometry turbocharging and flexible intake valve actuation. A direct measurement of the SOC is used for comparison by utilizing an in-cylinder pressure transducer and a CA encoder processing the cycle data in real time in reference set point. The controller utilized the crank angle for 50% burned mass fraction (CA50), calculated from the in-cylinder pressure data, as feedback to adjust the fuel injection timing. The controller was experimentally validated against engine data containing an engine load ramp at a constant engine speed. A stability analysis was not included.

Combustion control in partially premixed combustion (PPC) modes has been investigated by Tunestal and Lewander and Lewander. System identification methods were used to model the dynamics between the fuel injection and the combustion characteristics. Model-based control was then utilized to enable PPC to occur by maintaining a CA50 that was within a predefined region while controlling the injection timing and the injection duration to keep the indicated mean effective pressure (IMEP) and the ignition delay at the desired values. This control method utilized in-cylinder pressure measurements to obtain feedback on the CA50, the IMEP, and the ignition delay.

Wang developed a hybrid robust nonlinear model-based controller for a modern diesel engine operating under multiple combustion modes. The intake manifold pressure, the intake manifold oxygen fraction, and the exhaust manifold pressure were selected as the controlled system outputs. Sliding-mode controllers were developed for each combustion mode considered, and the controllers were switched by a designed supervisory controller. The controller was experimentally validated against engine data containing the EGR valve and VGT actuator step changes and the engine load ramps. The controller took into account the advanced combustion modes but did not control or estimate the combustion phasing.

Other studies have investigated control of advanced combustion modes including a paper by Husted et al. in which a proportional–integral controller was developed to control the combustion phasing of a diesel engine operating in pre-mixed diesel combustion to a
Several underlying estimators, which were previously developed by the present authors, are used in this effort and as such will be only briefly covered in this paper. The nonlinear controller focused on in this work presents a significant step forward in actually demonstrating control of the combustion timing in PCCI.

**Experimental setup**

A 2010 Cummins diesel engine outfitted with an electrohydraulic VVA system was utilized in this study. The Cummins ISB engine is a 6.7 l, 360 hp six-cylinder direct-injection diesel engine. As shown in Figure 2, the engine is an in-line six-cylinder configuration equipped with a Bosch common-rail fuel injection system with multi-pulse injection capability and a cooled EGR loop. The engine also has a VGT to boost the engine performance over the entire operating range as well as an electronic EGR valve to allow control of the fresh charge and EGR flows delivered to the cylinder.

The electrohydraulic VVA system, which was manufactured by Team Corporation and described in more detail by Modiyani et al. and Kocher et al. is capable of modifying the intake valve opening, the peak intake valve lift, and the intake valve closing (IVC) on a cylinder-independent cycle-to-cycle basis. The VVA system was used to alter the IVC timings in this work. The ability to control the intake valve actuation provides the capability to alter the ECR which affects the volumetric efficiency and the amount of piston-motion-induced compression.

The experimental engine data are acquired using a dSPACE system. The dSPACE system collects data from the engine electronic control module (ECM) such as the commanded fueling quantities and timings as well as the ECM sensor measurements. The engine is equipped with an open architecture ECM that allows direct read and write access to the memory locations at 100 Hz. The dSPACE system also collects data from additional temperature, pressure, flow, and emissions measurements instrumented on the engine test bed. The fresh-air mass flow rate is measured using a laminar flow element (LFE) device. The engine charge flow is calculated using the LFE fresh-air flow measurement and the measured EGR fraction. Emission gas analyzers are used to measure the composition of the exhaust gases as well as the concentration of carbon dioxide (CO₂) in the intake manifold. Cambustion nondispersive infrared fast CO₂ analyzers were utilized during this testing. The EGR fraction is computed using the intake and exhaust manifold CO₂ measurements. The intake manifold CO₂ is sampled in three locations and averaged to provide the best measurement average of the true intake manifold CO₂, as shown in Figure 2. A universal exhaust gas oxygen sensor is mounted in the exhaust pipe shortly after the turbine outlet, as shown in Figure 2.

**Figure 2.** Schematic diagram of a modern diesel engine.

CO₂: carbon dioxide; EGR: exhaust gas recirculation; IMT_TEMP: intake manifold temperature; Inj.: injection; CAC: charge air cooler; IVC: intake valve closing; ECR: effective compression ratio; EXH_PORT_TEMP: exhaust port temperatures; EGR_CLR_IN_T: exhaust gas recirculation cooler temperature in; EGR_CLR_OUT_T: exhaust gas recirculation cooler temperature out; TURB_IN_T: turbocharger temperature in; VGT: variable-geometry turbocharger; LFE: laminar flow element; DELTA_P_SENSOR: Delta pressure sensor; O₂: oxygen.


**PCCI combustion timing model**

As mentioned previously, one of the major challenges in using PCCI is achieving the proper timing of the combustion event. The controller developed in this work utilizes a combustion timing model for diesel PCCI developed by three of the present authors and co-workers.27 A short description of the model is provided below.

**PCCI combustion timing model development**

The time delay that exists between the SOIECM and the SOC is modeled as three distinct consecutive delays according to

\[
\text{SOI}_{ECM} + \tau_{elec} + \tau_{hyd} + \tau_{id} = \text{SOC}
\]

where \(\tau_{elec}\) is the delay present in the electrical system, \(\tau_{hyd}\) is the hydraulic delay present in the injector, and \(\tau_{id}\) is the ignition delay.

Analysis of the injector current signal reveals a difference between the SOIECM timing and when the injector current actually begins to rise. This average value is the electrical delay \(\tau_{elec}\) given by

\[
\tau_{elec} = -1.3^\circ\text{CA}
\]

This observed delay is due in part to an actual delay in the electrical subsystem as well as to an offset in top dead center (TDC). The TDC offset is due to a discrepancy between the TDC location reported by the engine’s crankshaft position sensor and the measured TDC provided by an AVL 365C position encoder. The AVL encoder has a resolution of 0.1° CA and allows precise knowledge of the true TDC. Since the effect of the TDC offset manifests itself in the injector current signal, it is included in the electrical delay. The observed electrical delay was constant in the CA domain.

The next delay in the system is a hydraulic delay in the injector, which is the time between when the injector current signal is enabled and when fuel droplets actually begin to exit from the injector nozzle. Examination of the injection rate-shape profiles allows characterization of the hydraulic delay for a variety of engine speed and load conditions. The hydraulic delay is essentially constant in the time domain at 0.3 ms. As such, in units of degrees CA, the hydraulic delay is given by

\[
\tau_{hyd} = 0.0018 N^\circ\text{CA}
\]

where \(N\) is the engine speed in revolutions per minute.

The ignition delay \(\tau_{id}\) is modeled using the Arrhenius-type correlation

\[
\tau_{id} = 0.051 F_{cyl}^{-1.14} \bar{P}^{-0.51} e^{2000/T}
\]

where \(\tau_{id}\) is in milliseconds, \(F_{cyl}\) is the in-cylinder oxygen fraction, and \(\bar{P}\) and \(T\) are the average in-cylinder pressure and average in-cylinder temperature respectively during the ignition delay period.27 The average pressure and average temperature over the ignition delay period from the SOI to the SOC can be approximated as

\[
P = \frac{1}{2} P_{in} V^{n_p}_{IVC} \left( V_{n_p}^{SOI + TDC/2} + V_{SOI}^{SOI} \right)
\]

\[
T = \frac{1}{2} P_{in} V^{n_p}_{IVC} \left( V_{n_p}^{SOI + 1} + V_{SOI}^{SOI + 1} \right)
\]

in which \(P_{in}\) is the intake manifold pressure, \(V_{IVC}\) is the cylinder volume at the IVC, \(V_{SOI}\) is the cylinder volume at the SOI, \(V_{n_p}\) is the polytropic coefficient, \(m_{charge}\) is the charge mass and \(R_{charge}\) is the gas constant for the charge air.

Combining the \(\tau_{id}\) model in equation (4) with the electrical delay and hydraulic delay in equation (2) and equation (3) respectively completes the PCCI combustion timing model first described in equation (1) and gives the expression for the SOC as

\[
\text{SOC} = -1.3 + 0.0018 N + 0.006 N (0.051 F_{cyl}^{-1.14} \bar{P}^{-0.51} e^{2000/T}) + \text{SOI}_{ECM}
\]

In which the SOC is in degrees CA, \(\bar{P}\) and \(T\) are calculated from equation (5) and equation (6) respectively, and \(N\) is in revolutions per minute. Additional details regarding the development of this model can be found in the paper by three of the present authors and co-workers.27 This model expresses the dependence of the combustion timing in PCCI on the in-cylinder conditions as well as the commanded fuel injection timing.

**PCCI combustion timing model experimental results**

The model detailed above was experimentally validated by three of the present authors and co-workers against 180 PCCI data points collected on the experimental engine test bed. SOIECM and IVC sweeps were performed at 12 nominal speed–load conditions, namely 271 N m, 203 N m, and 102 N m (200 ft lbf, 150 ft lbf, and 75 ft lbf), each at 2400 r/min, 2000 r/min, 1600 r/min, and 1200 r/min. The VGT position was varied to provide sufficient EGR to remain in a PCCI combustion mode. All data points presented in this study have a single main injection of fuel and have a carbon balance error of less than ±10%. (The carbon balance is the difference between the carbon in the intake versus the carbon in the exhaust and was evaluated on the basis of the CO2 measurements and the injected fuel mass.) Note that all timings are reported in degrees CA after top dead center (ATDC) of firing.

The PCCI combustion timing model in equation (7) maps from the SOIECM to the SOC, given knowledge of the total in-cylinder oxygen mass fraction (\(F_{in}\)), the average pressure (\(\bar{P}\)) and the average temperature (\(T\)) across the ignition delay, and the engine speed \(N\). The model predicts the SOC to within ±2° CA of the experimental values for all but three of the 180 data points, which represents 98%+ accuracy. The r.m.s.
error is 0.86° CA. Figure 3 shows the model and experimental SOC data in a scatter plot for comparison. The solid line is the 1:1 line and the dashed lines signify ±2° CA. The PCCI combustion timing model will be utilized to control the actual SOC in the following sections. As the timing model predicts the SOC to within ±2° CA, it is expected that the combustion timing controller should be able to maintain a similar level of accuracy. However, a portion of the observed error in the controlled SOC will be due to the uncertainty in the model.

PCCI combustion timing controller

The design of the PCCI combustion timing controller begins by examining the relationship between the control inputs and the desired SOC, as described in equation (7). The inputs to the control model include the SOI_{EGR}, the in-cylinder oxygen fraction, and the in-cylinder temperature and pressure during the ignition delay period. While examining the inputs, it is apparent that the inputs can be separated on the basis of the speed of the dynamics associated with the inputs. In this case, the in-cylinder oxygen fraction and the pressure and temperature during the ignition delay period are heavily dependent on the gas exchange processes, while the commanded SOI is essentially infinitely fast owing to the bandwidth of the fuel system. This separation of the dynamics allows a decoupled controller approach to be made. The slower controller will focus on controlling the dynamics associated with the gas exchange process, and the faster controller will focus on controlling the dynamics associated with the fuel injection process. Previous work by three of the present authors and co-workers has demonstrated that the in-cylinder oxygen fraction is extremely close to the intake manifold oxygen fraction. This is due to the small amount of residual exhaust gas that remains in the cylinder. Therefore, the assumption will be made to neglect the effect of the residual exhaust gas on the in-cylinder oxygen fraction, and the intake manifold oxygen fraction will be utilized directly in equation (7).

Previously developed and validated models of the gas exchange processes will be employed for the development of the slower controller which focuses on tracking the desired in-cylinder conditions. As a first step, only the in-cylinder oxygen fraction is considered as an input to the PCCI combustion timing controller. The variation in the in-cylinder temperature and pressure during the ignition delay period will not be directly controlled but instead will be considered as a disturbance to the system.

The intake manifold oxygen fraction dynamics are described by

$$F_{\text{in}} = \frac{P_{\text{in}}}{P_{\text{c}}} \left( F_{\text{amb}} W_c + F_{\text{en}} W_{\text{EGR}} - F_{\text{im}} W_c \right)$$  \(8\)

where $F_{\text{in}}$ is the fraction of oxygen in the intake manifold, $R$ is the universal gas constant, $T_{\text{in}}, P_{\text{in}},$ and $V_{\text{in}}$ are the temperature, pressure, and volume respectively in the intake manifold, $W_c$ is the flow through the compressor, $W_{\text{EGR}}$ is the EGR flow, and $W_c$ is the flow into the engine cylinder. The development of this model for the intake manifold oxygen dynamics as well as validation of this model over a wide speed range (1000–2300 r/min) and load range (200–522 N m) have been detailed previously by three of the present authors and co-workers.

Examination of equation (8) reveals the importance of the flows $W_c$ and $W_{\text{EGR}}$ into the intake manifold, as well as the flow $W_c$ leaving the intake manifold. The traditional control actuators for the gas exchange process include the EGR valve position and the VGT position. However, PCCI combustion modes typically require large quantities of EGR, resulting in an EGR valve position of 100% under all operating conditions. Therefore, the VGT position will be the controlling actuator for the intake manifold oxygen fraction dynamics. The VGT position will have a direct impact on the flow ($W_c$) through the turbocharger turbine and, in turn, will control the compressor flow ($W_c$). As such, recording the dynamics of the turbocharger is essential to properly controlling the VGT position such that the desired intake manifold oxygen fraction is achieved.

The VGT turbine is modeled using analytical functions based upon turbine maps provided by the turbocharger manufacturer to determine the flow through the turbine. The analytical functions were derived and validated by three of the present authors and co-workers and allow the turbine speed and mass flow to be found independently of the manufacturer’s maps, reducing the computation time for real-time estimation and control. Turbine maps generally express the turbine speed and mass flow in terms of reduced quantities to account for the inlet conditions.
\[ W_{t,\text{red}} = \frac{W_t \sqrt{T_{\text{em}}}}{P_{\text{em}}} \quad (9) \]

\[ \omega_{tc, \text{red}} = \frac{\omega_{tc}}{\sqrt{T_{\text{em}}}} \quad (10) \]

where \( W_{t,\text{red}} \) is the reduced turbine mass flow, \( \omega_{tc,\text{red}} \) is the reduced turbine speed, and \( T_{\text{em}} \) and \( P_{\text{em}} \) are the turbine inlet temperature and turbine inlet pressure respectively.

The reduced turbine flow (\( W_{t,\text{red}} \)) is calculated through an analytical function using the pressure ratio (\( P_R \)) across the turbine and the turbocharger shaft speed \( \omega_{tc,\text{red}} \) according to

\[ W_{t,\text{red}} = \frac{\pi \eta_t^2 (\gamma_{R_{\text{exh}}})^{1/2} [b_1 + b_2 \sqrt{2\gamma_{R_{\text{exh}}} (1 - \frac{1}{\gamma_{R_{\text{exh}}}}) \left( \frac{\pi}{60} \frac{d_{\text{amb,red}}}{d_{\text{amb,red}}} \right)^{2(1 - 1/\gamma_{R_{\text{exh}}})}]}{4 \sqrt{\gamma_{R_{\text{exh}}} (1 - \frac{1}{\gamma_{R_{\text{exh}}}})}} \]

Here, \( R_{\text{exh}} \) and \( c_p,\text{exh} \) are assumed to be constant. The values for \( b_1 \) and \( \alpha_i \), which are found via the least-squares method using the manufacturer’s turbocharger maps, are listed in Table 1. The reduction of the turbocharger maps has been covered in detail previously by three of the present authors and co-workers.31 Note that the three inputs to equation (11) are \( \omega_{tc,\text{red}} \), \( P_R \), and \( X_{\text{VGT}} \) (the VGT position) and result in reduced mass flow. This reduced mass flow can then be used in equation (9) to back out the actual mass flow through the turbine.

Equations (9) to (11) represent the link between the VGT position, which will be used as a control input for the PCCI controller, and the turbine flow.

The turbine flow is related to the turbine power \( P_{\text{turb}} \) by

\[ P_{\text{turb}} = W_{t,\text{p,exh}} \eta_{\text{turb}} T_{\text{em}} \left( 1 - \frac{P_{\text{amb}}}{P_{\text{em}}} \right)^{\gamma_{\text{exh}} - 1} \]  

\[ (12) \]

The turbine and compressor powers are related by considering the turbocharger shaft dynamics according to

\[ \dot{\omega}_{tc} = \frac{P_{\text{turb}} - P_{\text{comp}}}{I_{\text{turb}} \omega_{tc}} \quad (13) \]

where \( \omega_{tc} \) is the turbocharger shaft speed, \( I_{\text{turb}} \) is the moment of inertia of the turbocharger, \( P_{\text{turb}} \) is the turbine power, and \( P_{\text{comp}} \) is the compressor power.

It is assumed that the turbocharger shaft is in the steady state. While this simplifying assumption could induce minor errors, the steady-state approximation is reasonable, given the large lumped volume between the turbocharger compressor outlet and the intake manifold. Furthermore, as will be shown in a subsequent section, even given this assumption the controller (which is derived from this, and other, models) is shown to perform quite well on an experimental test bed.

Applying the steady-state assumption to the turbocharger shaft dynamics shown in equation (13), the compressor power (\( P_{\text{comp}} \)) can be assumed to be equal to the turbine power (\( P_{\text{turb}} \)). The compressor power (\( P_{\text{comp}} \)) can then be utilized to calculate the compressor flow (\( W_c \)) using

\[ P_{\text{comp}} = \frac{W_{c,\text{p,exh}} T_{\text{amb}}}{\eta_{\text{comp}}} \left( \frac{P_{\text{amb}}}{P_{\text{em}}} \right)^{\gamma_{\text{exh}} - 1} \] 

\[ (14) \]

To simplify the notation, the following substitutions are made:

\[ W_c = k_{\text{comp}} P_{\text{comp}} \]

\[ (15) \]

\[ k_{\text{comp}} = \frac{c_{p,\text{exh}} T_{\text{amb}}}{\eta_{\text{comp}}} \left( \frac{P_{\text{amb}}}{P_{\text{em}}} \right)^{\gamma_{\text{exh}} - 1} \] 

\[ (16) \]

\[ P_{\text{turb}} = k_{\text{turb}} W_1 \]

\[ (17) \]

\[ k_{\text{turb}} = \frac{c_{p,\text{exh}} T_{\text{em}}}{\eta_{\text{turb}}} \left( 1 - \left( \frac{P_{\text{amb}}}{P_{\text{em}}} \right)^{\gamma_{\text{exh}} - 1} \right) \] 

\[ (18) \]

\[ W_c = k_{\text{comp}} k_{\text{turb}} W_1 \]

\[ (19) \]

\[ k_{\text{Fim}} = \frac{R_{\text{tc}} T_{\text{im}}}{P_{\text{tc}} V_{\text{im}}} \]

\[ (20) \]

The \( W_{\text{EGR}} \) term in equation (8) will be treated as a disturbance to the system since the main control actuator, namely the EGR valve position, is saturated at its maximum position. The value of \( W_{\text{EGR}} \) is provided by an EGR flow estimator developed by Kocher32 and is dependent on the pressure drop across the engine. The intake manifold oxygen fraction dynamics are now cast in the state-space equivalent form

\[ \dot{F}_{\text{im}} = k_{\text{Fim}} F_{\text{amb}} k_{\text{comp}} k_{\text{turb}} W_1 + k_{\text{Fim}} F_{\text{em}} W_{\text{EGR}} - F_{\text{im}} W_e \]

\[ (21) \]

Equation (21) can be expressed as the linear parameter-varying form

\[ \dot{x} = A(\rho)x + B(\rho)u + G(\rho) \]

\[ (22) \]

in which

\[ A(\rho) = -k_{\text{Fim}} W_e \]

\[ (23) \]

\[ B(\rho) = k_{\text{Fim}} F_{\text{amb}} k_{\text{comp}} k_{\text{turb}} \]

\[ (24) \]

\[ G(\rho) = k_{\text{Fim}} F_{\text{em}} W_{\text{EGR}} \]

\[ (25) \]

where the system parameters are expressed in \( \rho \), the state \( x \) is the intake manifold oxygen fraction (\( F_{\text{im}} \)), and the input \( u \) is the turbine flow (\( W_t \)) which is directly controllable by adjusting the VGT position.

With the system dynamics defined, a control law may be selected to stabilize the system and to provide reference tracking of the desired \( F_{\text{im}} \) values. The selected control law is given by
where \( r_f \) is the filtered version of the \( F_{im} \) reference command \( r \) after it is filtered by the system dynamics and is given by

\[
\dot{r}_f = A r_f - A r - k_{F_{im}} W_e r_f + k_{F_{in}} W_e r
\]

(27)

The usage of the matrix \( A \) is a suitable choice since it reflects the dynamics of the physical system.

Substitution of the control law in equation (26) into the system dynamics equation (21) yields the closed-loop expression

\[
\dot{x} = A x + B [K(r_f - x) + L] + G
\]

(28)

With the control law selected for the slower gas exchange dynamics, fixing the values of \( F_{im} \), \( P \), and \( T \), the SOI_{ECM} can be determined on the basis of the engine speed and the desired SOC. The SOI_{ECM} is calculated using

\[
\text{SOI}_{ECM} = \text{SOC} - 1.3 - 0.0018 N - 0.006 N (0.051 F_{im}^{1.4} P^{-0.51} e^{2100/T})
\]

(29)

A high-level graphical representation of the controller structure is shown in Figure 4. The PCCI controller receives commands for the desired SOC and \( F_{im} \) and utilizes the intake manifold oxygen controller represented by equation (28) to determine the required turbine flow input. The VGT position is then calculated using equations (9) to (11). The required SOI will be determined on the basis of equation (29) and the current in-cylinder conditions.

Figure 5 shows the PCCI controller structure and includes the additional models and estimators utilized in the control. Underlying models of the gas exchange processes including estimates of the volumetric efficiency, the EGR flow, the turbine flow, and the intake manifold oxygen fraction provide inputs to the PCCI controller which are utilized in the \( F_{im} \) controller as well as in the SOI calculation.

**PCCI combustion timing controller stability**

With the closed-loop system dynamics and the reference filter dynamics defined, the stability of the closed-loop system may be analyzed by examining the error dynamics given by

\[
e = r_f - x
\]

(30)

\[
\dot{e} = \dot{r}_f - \dot{x}
\]

(31)

Substitution of the expressions for \( \dot{r}_f \) and \( \dot{x} \) into equation (31) yields the equation

\[
\dot{e} = A r_f - A r - A x - B [K(r_f - x) + L] - G
\]

(32)

After simplification, the expression for \( \dot{e} \) is

\[
\dot{e} = (A - BK)e - Ar - BL - G
\]

(33)

The gain \( L \) may be selected to negate the error dynamics associated with the original reference.
command \( r \) and the disturbance due to the \( W_{EGR} \) term and is given by

\[
L = \frac{-AR - G}{B}
\]

\[
L = \frac{W_r - F_{em}W_{EGR}}{k_{comp}k_{turb}F_{amb}}
\]  \hspace{1cm} (34)

\hspace{1cm} (35)

This selection of the gain \( L \) will reduce the error dynamics to

\[
\dot{e} = (A - BK)e
\]

(36)

The selection of the gain \( K \) will be determined on the basis of the experimental controller performance tuning and a Lyapunov analysis will be performed to assess the stability. A Lyapunov function \( V \) is chosen as

\[
V = \frac{1}{2}e^2
\]

(37)

The derivative of the Lyapunov function is then calculated as

\[
\dot{V} = e\dot{e}
\]

(38)

Substituting the expression for \( \dot{e} \) in equation (36) into equation (38) yields the equation

\[
\dot{V} = e(A - BK)e
\]

(39)

After substituting for \( A \) and \( B \) in equation (39), the expression can be simplified to

\[
\dot{V} = (-k_{Fim}W_{e} - k_{Fim}k_{comp}k_{turb}F_{amb}K)e^2
\]

(40)

To demonstrate the stability, the Lyapunov function \( V \) needs to be positive definite and the derivative of the Lyapunov function \( \dot{V} \) needs to be negative definite. Examination of equation (40) shows that the error dynamics will be stable and \( \dot{V} \) will be negative definite as long as \( K > 0 \). The difficulty in selecting the gain \( K \) arises when evaluating the controller performance dynamically and assessing the desired performance and robustness tradeoffs associated with the controller design. While the choice of controller gain will influence the transient response, the steady-state error, and the robustness over the operating conditions examined in this study, the error dynamics are expected to be stable and to converge asymptotically for any positive gain value (as shown in the analysis). In order to validate the control strategy experimentally, the gain was tuned to 3000 in order to give a good balance between the transient response, the error, and the robustness over different operating conditions. Care was taken to avoid engine damage that might result from misfire or overly advanced combustion timing.

**Experimental results for the PCCI combustion timing controller**

The PCCI combustion timing controller developed was implemented using a dSPACE control system. The controller was then tested in multiple operating conditions to demonstrate its performance. A variety of transitions were investigated including steps in the intake manifold oxygen fraction, steps in the commanded SOC, steps in the VC, and steps in the fueling.

**Performance during commanded intake manifold oxygen steps**

It is essential that the controller tracks changes in the desired intake manifold oxygen fraction while also tracking the desired SOC.

Figure 6 shows the controller operating at an engine speed of 1600 r/min and an engine torque of 140 ft lb. The desired SOC was fixed at –5° CA ATDC, and the \( F_{im} \) command was stepped from 20% oxygen to 16% oxygen. In Figure 6, the upper left-hand plot shows the oxygen fraction in both the intake manifold and the exhaust manifold. The solid black curve is the reference filtered commanded intake manifold oxygen fraction. The dashed red curve is the estimated intake manifold oxygen fraction, and the dot-dashed blue curve is the measured exhaust oxygen fraction. The manifold oxygen fraction plot shows the step change in the desired intake manifold oxygen fraction and the estimated intake manifold oxygen fraction tracking the reference command. The estimated value of \( F_{im} \) tracks the desired oxygen fraction to within 1% oxygen as desired. The upper right-hand plot shows the desired SOC as a solid black line. The dashed red line is the measured SOC based upon the in-cylinder pressure trace and the blue dot-dashed curve is the commanded SOI. Figure 6 shows that the SOC is controlled to the desired SOC to within 1° CA while \( F_{im} \) is being step changed. The commanded SOI is adjusted by the controller to compensate for the change in \( F_{im} \), as evidenced by examining the SOI command during the \( F_{im} \) step changes. The bottom left-hand plot shows the gas exchange actuator positions. The VGT position is shown as a solid black curve. The VGT position is adjusted to control the estimated \( F_{im} \) to the desired \( F_{im} \). The EGR valve position is shown as a dashed red line at 100% open. As a large amount of EGR is required to ensure sufficiently long ignition delays in PCCI, the EGR valve is required to be fully open during all tests. The IVC timing is shown as a blue dot-dashed line and is fixed at 565° CA ATDC, which is the nominal IVC timing. The bottom right-hand plot shows the commanded turbine flow as a solid black curve. The dashed red curve shows the actual turbine flow as calculated by the turbocharger analytical functions described by three of the present authors and co-workers. The calculated turbine flow is controlled to the commanded turbine flow by adjusting the VGT position. A larger VGT position, reported as the percentage closed, allows less flow to pass through the turbine when compared with a smaller VGT position, allowing the turbine flow to be controlled through adjustment of the VGT position. Note
that, when a smaller amount of intake manifold oxygen is required, the VGT closes, thereby reducing the turbine flow and providing a higher EGR flow and the desired intake manifold oxygen level.

Figure 7 shows the controller operating at an engine speed of 1200 r/min and an engine torque of 140 ft lb. The desired SOC was held constant at 0°C176CA ATDC, and the Fim command was stepped between 20% oxygen and 16% oxygen. The IVC timing was held constant at 565°C176CA ATDC. The effects on the system at this lower engine speed are handled by the controller. In both of these cases, the desired Fim and the desired SOC are achieved.

Performance during commanded SOC steps
The previous results demonstrated that different SOC values could be tracked during steps in the desired intake manifold oxygen fraction. Next, the controller’s ability to track steps in the desired SOC is evaluated.

Figure 8 shows the controller operating at an engine speed of 1600 r/min and an engine torque of 140 ft lb. The desired SOC was held constant at 0°C176CA ATDC, and the Fim command was stepped between 20% oxygen and 16% oxygen. The IVC timing was held constant at 565°C CA ATDC. The effects on the system at this lower engine speed are handled by the controller. In both of these cases, the desired Fim and the desired SOC are achieved.

Performance during commanded SOC steps
The previous results demonstrated that different SOC values could be tracked during steps in the desired intake manifold oxygen fraction. Next, the controller’s ability to track steps in the desired SOC is evaluated.

Figure 9 shows the controller operating at an engine speed of 1200 r/min and an engine torque of 140 ft lb. The desired SOC was stepped, and the Fim command was held constant at 16% oxygen in Figure 9. The IVC timing was held constant at 565°C CA ATDC.

Performance during IVC steps
The valvetrain flexibility is another key enabler of PCCI, and changes in the valve timings will have a direct impact on the ECR of the engine and the volumetric efficiency of the engine. In this section, steps in the IVC are considered.
Figure 7. PCCI combustion timing controller (SOC, 0° CA ATDC; F_{in}, stepped; IVC, 565° CA ATDC at 1200 r/min and 140 ft lbf). O_2: oxygen; Cmd: command; SOC: start of combustion; atdc: after top dead center; SOI: start of injection; EGR: exhaust gas recirculation; VGT: variable-geometry turbocharger; IVC: intake valve closing; Wt: turbine flow.

Figure 8. PCCI combustion timing controller (SOC, stepped; F_{in} = 20%; IVC, 565° CA ATDC at 1600 r/min and 140 ft lbf). O_2: oxygen; Cmd: command; SOC: start of combustion; atdc: after top dead center; SOI: start of injection; EGR: exhaust gas recirculation; VGT: variable-geometry turbocharger; IVC: intake valve closing; Wt: turbine flow.
Figure 10 shows the controller operating at an engine speed of 1200 r/min and an engine torque of 140 ft lbf. The desired SOC was held constant at –5° CA ATDC and the $F_{im}$ command was held constant at 18% oxygen. The IVC timing was stepped from 565° CA ATDC to 535° CA ATDC and then back to 565° CA ATDC. This corresponds to a change in the ECR from the nominal ECR of 18.7 at the conventional IVC timing (565° CA ATDC) to a reduced ECR of 15.9 at the early IVC timing (535° CA ATDC). In Figure 10, the manifold oxygen fraction plot shows the desired intake manifold oxygen fraction and the estimated intake manifold oxygen fraction tracking the reference command. The estimated value of $F_{im}$ tracks the desired oxygen fraction despite a disturbance to the system in the form of a change in the ECR. The upper right-hand plot shows control of the SOC to the desired SOC while $F_{im}$ is held constant. The commanded SOI is adjusted to control the desired SOC owing to the changes in the in-cylinder pressure and in the temperature. The bottom left-hand plot shows the adjustment of the VGT position to control the estimated $F_{im}$ to the desired $F_{im}$. The EGR valve position is shown as a dashed red line at 100% open. The IVC timing is shown as a dot-dashed blue curve and is stepped between 565° CA ATDC and 535° CA ATDC. The bottom right-hand plot shows that the actual turbine flow is controlled to the commanded turbine flow by adjusting the VGT position. The controller is capable of controlling the desired $F_{im}$ and the desired SOC during the IVC step change.

**Performance during fueling and load steps**

While changes in the desired oxygen levels, the SOC timing, and the IVC timing are commonly encountered particularly in PCCI, it is also essential to consider changes in the engine load conditions.

Figure 11 shows the controller performance during a fueling step change. The desired SOC was held constant at –5° CA ATDC, and the $F_{im}$ command was held constant at 18% oxygen. The IVC timing was held constant at 535° CA ATDC. While the engine speed was held constant at 1600 r/min, the fueling command was stepped from 19 mg/stroke to 39 mg/stroke at 12 s and then back to 19 mg/stroke at 26 s. This fueling command step results in a load step from 80 ft lbf to 200 ft lbf and back. Figure 11 demonstrates the controller’s...
Figure 10. PCCI combustion timing controller (SOC, –5° CA ATDC; \( F_{im} = 16\% \); IVC, stepped at 1200 r/min and 140 ft lbf). O\(_2\): oxygen; Cmd: command; SOC: start of combustion; ATDC: after top dead center; SOI: start of injection; EGR: exhaust gas recirculation; VGT: variable-geometry turbocharger; IVC: intake valve closing. Wt: turbine flow.

Figure 11. PCCI combustion timing controller (SOC, –5° CA ATDC; \( F_{im} = 18\% \); IVC, 565° CA ATDC at 1600 r/min and stepped fueling). O\(_2\): oxygen; Cmd: command; SOC: start of combustion; ATDC: after top dead center; SOI: start of injection; EGR: exhaust gas recirculation; VGT: variable-geometry turbocharger; IVC: intake valve closing. Wt: turbine flow.
ability to control the desired $F_{im}$ and SOC under engine fueling step changes.

**Performance during combined changes**

The previous results indicate that independent changes in the commands and the disturbances to the system are handled well with this control strategy. Next, the controller’s performance is considered during simultaneous changes in the commands and the disturbances.

Figure 12 shows the controller operating at an engine speed of 1600 r/min and an engine torque of 140 ft lbf. The desired $F_{im}$ was held constant at 18% oxygen and the desired SOC command was stepped. The IVC timing was held constant at 535° CA ATDC. Recall that changes in the IVC from the nominal value (565° CA ATDC) serve as a disturbance to this system by altering the ECR and the volumetric efficiency. In Figure 12, the controller’s tracking performance is similar to the previously described cases with $F_{im}$ being controlled to within 1% oxygen and the SOC being controlled to within 1° CA even with a disturbance to the system in the form of an early IVC timing.

Similar results are observed at the operating point for 1200 r/min and 140 ft lbf. Figure 13 shows the controller operating at an engine speed of 1200 r/min and an engine torque of 140 ft lbf. The desired SOC was held constant at –5° CA ATDC and the $F_{im}$ command was stepped between 20% oxygen and 16% oxygen. The IVC timing was held constant at 535° CA ATDC. In Figure 13, the $F_{im}$ controller is unable to drive the estimated $F_{im}$ completely to the desired $F_{im}$ command under the 20% oxygen command condition. In this case, the controller still achieves an error of less than 1% oxygen but is unable to reduce the error any further. The reason for the degraded controller performance is due to the uncertainty in the models used in the nonlinear model-based controller. The controller relies on the accuracy of the models in generating the controller commands, and inaccuracy of these models leads to degradation in the controller performance.

**Conclusions**

The controller developed in this work was shown to be capable of accurately controlling the combustion phasing of a modern diesel engine operating in PCCI. The controller’s stability was demonstrated through Lyapunov analysis, and the controller’s functionality was experimentally validated at multiple operating
conditions. The results demonstrate that this controller is capable of controlling the SOC to within 1° CA of a desired value while also controlling the intake oxygen fraction to within 1% of a desired oxygen fraction value.

Future work

In order to enable PCCI to occur and thereby to reduce emissions, VVA capabilities can also be utilized. Future work will explore the combined actuation of valve timings together with VGT and EGR settings for more accurate tracking of desired trajectories and faster response times. The models employed in this effort will also be used to explore transitions between PCCI and conventional diesel combustion in future studies.

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Declaration of conflict of interest

The authors declare that there is no conflict of interest.

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Appendix I

Notation

\( F_{\text{amb}} \) ambient air oxygen fraction
\( F_{\text{ci}} \) in-cylinder oxygen fraction
\( F_{\text{em}} \) exhaust manifold oxygen fraction
\( F_{int} \) intake manifold oxygen fraction  
\( m_{charge} \) charge mass  
\( N \) speed of the engine  
\( \mathcal{P} \) average pressure during ignition delay  
\( P_{amb} \) ambient pressure  
\( P_{comp} \) compressor power  
\( P_{em} \) exhaust manifold pressure  
\( P_{im} \) intake manifold pressure  
\( P_{turb} \) turbine power  
\( R \) universal gas constant  
\( \bar{T} \) average temperature during ignition delay  
\( T_{amb} \) ambient temperature  
\( T_{em} \) exhaust manifold temperature  
\( T_{im} \) intake manifold temperature  
\( V_{im} \) intake manifold volume  
\( V_{IVC} \) volume at intake valve closing  
\( V_{(SOI+ TDC)/2} \) volume at midpoint between the start of injection and top dead center  
\( W_{c} \) fresh air flow from the compressor  
\( W_{e} \) charge flow from the intake manifold to the cylinders  
\( W_{EGR} \) exhaust gas recirculation flow  
\( W_l \) flow from the exhaust manifold through the turbine  
\( W_{t,red} \) reduced turbine mass flow  
\( X_{VGT} \) position of the variable-geometry turbocharger  
\( \eta_{comp} \) efficiency of the compressor  
\( \eta_{turb} \) efficiency of the turbine  
\( \tau_{elec} \) electrical delay  
\( \tau_{hyd} \) hydraulic delay  
\( \tau_{id} \) ignition delay  
\( \omega_{tc} \) turbocharger shaft speed  
\( \omega_{tc,\text{red}} \) reduced turbocharger shaft speed  

**Abbreviations**  
SOC start of combustion  
SOI\(_{ECM} \) start of injection commanded by the engine electronic control module  
PR\(_{t} \) pressure ratio across the turbine